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Received 28 June 2018
Revised 15 January 2019
Accepted 27 February 2019

Efficiency, stability, and government regulation of risk-sharing financial networks

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Abstract

Purpose – By applying models of social and economic networks to financial institutions, the purpose of this paper is to address the issues of how policy makers can promote financial network stability and social efficiency.
Design/methodology/approach – The authors characterize the decentralized network formation of financial institutions in three stages through which institutions choose to become member banks connected to a central bank, bank-holding company subsidiaries or non-banks. Financial institutions choose one of the three roles in an endogenous process by considering the effects of sharing shocks among the members of the network. In the model, there is a social-welfare-maximizing government regulator at the center of the network.
Findings – The authors show that the stable equilibrium network is not always the efficient network, so the central authority must use policy instruments to ensure that the stable equilibrium network is as close as possible to the efficient network.
Research limitations/implications – To obtain the theoretical results, the authors make assumptions about the utility function and risk aversion of a financial institution, as well as about the costs of network formation. These assumptions might need to be relaxed to bring the model closer to real-world institutions.
Practical implications – The results suggest that regulators must try to set their policy variables to make the efficient network as close as possible to the stable network.
Originality/value – The contribution is to incorporate concepts from social network theory into the modeling of financial networks. The results may be of use to regulators in maintaining the stability of the financial system.

Keywords Banks, Efficiency, Stability, Financial networks, Regulatory policy

Paper type Research paper

1. Introduction

At the beginning of 2008, the five largest independent investment banks on *Wall Street* were Goldman Sachs, Morgan Stanley, Merrill Lynch, Lehman Brothers and Bear Stearns. By the end of that year, as the financial crisis proceeded, none of them retained that status. Four of them became affiliated in one way or another with commercial banks and their holding companies. Those connections, in turn, created stronger links to the banking regulatory system of the USA. Lehman Brothers went through bankruptcy, but the surviving parts of their North American operations became part of Barclay's Bank, which has retail and commercial banking operations in the USA. In this paper, we analyze the process by which financial institutions become, or choose not to become, part of the financial network connected to the regulatory system.

It is generally agreed that an important role of government regulators is to maintain financial stability and ameliorate the effects of financial crises, reducing the real costs that can grow as a problem spreads across the economy. Therefore, understanding financial

JEL Classification — G18, G21, G29

The authors would like to thank Dr William V. Rapp and the Leir Foundation for financial support for an early version of this paper.

Managerial Finance
Vol. 45 No. 6, 2019
pp. 760-780
© Emerald Publishing Limited
0307-4358
DOI 10.1108/MF-06-2018-0287

system linkages is critical to developing regulatory policy actions. Policy makers have long been concerned about the risk of a singular financial event creating a more general crisis. The "Great Depression" of the 1930s was an extreme example of a financial crisis that had a negative impact on real economic growth (Friedman and Schwartz, 1963; Bernanke, 1983). The Federal Reserve Bank's 1998 bailout of the hedge fund Long-Term Capital Management was motivated by concerns about the potential that a large hedge fund failure could destabilize the whole financial system (Dungey *et al.*, 2006; Kabir and Hassan, 2005). The financial crisis that began in 2008 is another good example, having required bailouts of large institutions and even countries by central banks around the world.

Modern financial networks are characterized by a very high degree of interdependence. The links between the financial regulators (FRs), regulated member banks (MBs), bank-holding company subsidiaries and non-bank (NB) financial institutions (often referred to as the shadow banking system) allow them to share risks. Banks often have contingent claims on each other which may be destabilizing for the system and challenging for regulators (Acharya *et al.*, 2017). The benefits of risk sharing are closely related to the problem of contagion. Individual bank diversification may not lead to superior performance as banks continue to play a special role due to information asymmetries (Acharya *et al.*, 2006; Gande and Saunders, 2012). As shown by Gallegati *et al.* (2008), linkages and risk sharing may be helpful in good economic times, but can be detrimental in a weakening economy. While bank regulation and especially accurate financial reporting may enhance financial stability (Acharya and Ryan, 2016), this is likely to be only part of the regulatory solution.

In the USA, financial regulatory functions are divided among several agencies whose roles are changing as a result of the Dodd-Frank Act (referred to in the following as Dodd-Frank)[1]. In our simplified model, the FR plays many of these roles and so represents the Federal Deposit Insurance Corporation plus the Federal Reserve and the Controller of the Currency. The FR's goal is to maintain financial market stability[2]. MBs establish direct links with the FR that provide for costly oversight and regulation. These MBs' costs are offset by the opportunity to accept deposits from the general public, to offer government-backed deposit insurance and to rely on the FR as a lender of last resort.

An MB may be part of a larger corporate entity known as a bank-holding company. JPMorgan Chase Bank, N.A., an MB, is part of JPMorgan Chase & Co., which is a bank-holding company, under which are other NB corporate entities that we generally characterize as bank-holding company subsidiaries (BHCS). One of JPMorgan Chase's important NB subsidiaries is J.P. Morgan Securities, an SEC-regulated broker dealer, commonly known as an investment bank, which engages in brokerage, portfolio management and other non-banking financial services, in addition to investment banking. While the Federal Reserve maintains regulatory oversight of bank-holding companies, the Fed has very limited control over the non-BHCS.

In addition to the NB financial corporations that are BHCS, there are many NB that are independent and without any regulatory oversight by the Federal Reserve or FDIC. Some, like TD Ameritrade, are SEC-regulated broker dealers, while others, like Soros Fund Management, are unregulated hedge funds. Insurance companies, mortgage companies, venture capital funds and many other important financial market participants are NB financial corporations. While the connections between MBs and the FR and between bank-holding companies and their subsidiaries are public and explicit, there are many other forms of risk sharing that financial firms use. Loan syndication agreements, credit default swaps and other derivative instruments are common contractual forms for risk sharing and may be used to link any of these entities. Real-world financial networks are extremely complicated and difficult to observe. In our current model, we consider only the membership and ownership relationships mentioned above. Thus, our financial networks are more sophisticated than in much recent literature, but are still highly simplified[3].

We assume here that financial institutions are risk averse[4] and create risk-sharing links to reduce their risk. To motivate the formation of financial networks, we posit that the costs of establishing links are offset by the potential benefits of risk sharing[5]. The hierarchical financial network that we observe is the result of strategic interactions among financial institutions. The formation of interrelationships is done before the realization of idiosyncratic and/or systemic shocks and results from a combination of top-down policy actions of the FR and bottom-up utility maximizing behavior from the individual financial institutions. Conceptually, the idiosyncratic shocks come from credit surprises from an institution's own portfolio. Each institution has a known distribution of future loan and credit losses, but the realized losses are unpredictable. The systemic shocks are based on macroeconomic events and may be realized as price shocks on assets that are commonly held. For example, if all institutions were holding their capital as unhedged Treasury bonds, then an unexpected general increase in interest rates would represent a negative systemic shock to their capital stocks[6].

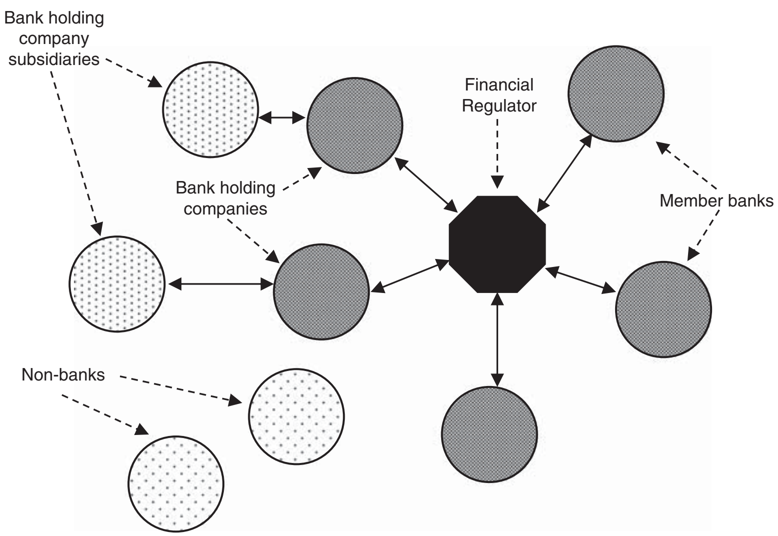
The contribution of this paper is threefold. We extend the traditional finance literature by developing a model that allows for endogenous financial network formation, based on social networking models. We characterize how financial institutions choose to become member banks, bank-holding company subsidiaries and non-bank. We present examples from the 2007–2009 Great Recession throughout the text. Second, we extend the social networking literature by incorporating an asymmetric player, a benevolent FR, which regulates member banks in such a way as to efficiently maximize social welfare[7]. The FR in this model chooses recognizable policy variables that drive the financial institutions' decentralized decision making and promote development of a socially optimal financial system. Third, by incorporating risk aversion and the cost of forming or unilaterally breaking a link, we show that the *ex ante* risk-sharing financial network of the MBs and BHCS is stable and can lower the investment risks of all the members of the network. Links between corporations are bilateral and must be pairwise stable. Links between subsidiaries of the same holding company may generate positive externalities by allowing for risk sharing through indirect connections. We describe a three-stage network formation game where in the first two stages roles are chosen and links are established, and then in the third stage shocks are realized. In our stylized model, any realized shocks must be shared across linked institutions. We characterize of the structure of stable networks where before any shocks are realized, no financial institution has any incentive to change position.

The asymmetric role of the FR gives our financial network a "star" shape with the FR at the center of the network, with member banks in the nearby periphery, and NB in the extreme positions (see Figure 1). An interesting feature is that otherwise identical financial institutions can choose different roles and therefore have different risk-sharing outcomes.

The rest of the paper is organized as follows. In Section 2, we review the literature on financial networks. In Section 3, we define the elements of the model and develop the benchmark model of risk sharing. In Section 4, we analyze the endogenous *ex ante* risk-sharing network and discuss several possibilities for network formation. We also examine the stability conditions of the *ex ante* risk-sharing network. In Section 5, we examine the welfare comparisons between efficient and equilibrium networks. We conclude in Section 6 with some policy implications, and list some directions that we would like to explore for future research. A numerical example of the model is presented in Appendix 1 along with some corollary results and boundary condition analysis.

2. Literature review

There is a substantial literature focused on understanding financial market structure to gain insights into the development of financial crises and contagion. Ho and Saunders (1980) set the stage by describing the instability of financial networks that lead to explosive and



Note: The network consists of the financial regulator, member banks that are connected directly to the financial regulator, bank holding company subsidiaries that are connected to member banks and non-banks that are not connected to any other institutions

Figure 1. Network example

catastrophic failures of banks. The seminal paper by Diamond and Dybvig (1983) developed a three-period model where fixed liability deposits, costly liquidation of some assets and exogenous shocks can lead to bank runs. They suggest that deposit insurance eliminates the possibility of bank runs by removing the incentive for depositors to line up to claim their deposits. With multiple equilibria, they attribute the choice of equilibrium to "sun spots" or random chance. Subsequent empirical research has suggested that bank runs are related to the state of the business cycle (Gorton, 1988). Though the sequential service constraint (first-come, first-served) and the "sun spots" equilibrium have been criticized, the Diamond and Dybvig model has been the basis for numerous subsequent papers, notably by Allen and Gale (1998, 2000, 2005, 2008).

Allen and Gale (1998) extend Diamond and Dybvig (1983) model to incorporate a "business cycle" risk in the long-term asset. When negative information about the returns of the long-term asset is revealed, it may cause a crisis. Instead of using the sequential service constraint, they replace it with an equal sharing among those claiming deposits early and a similar sharing among late claimants. In their model with costly liquidation of the long-term asset, they demonstrate that the free market equilibrium reduces the depositor's consumption and does not achieve a "first-best allocation." The costly liquidation of the long-term asset provides an incentive for regulatory intervention, but the FR is exogenous to the model.

Allen and Gale (2000) extend the representative bank model above to an economy consisting of a number of regions. When each bank/region has an individual shock, they can use inter-regional deposit holdings to insure against a local shock. This system of deposits works well as long as there is sufficient liquidity within the whole financial system, and as long as there are complete markets, so that banks/regions that need liquidity can find the banks/regions that have excess liquidity. In the event that markets for deposits are incomplete such that banks needing liquidity cannot find the banks with excess liquidity, and also in the event that aggregate liquidity is insufficient to meet depositors' needs, there may be a cascading effect where banks are forced into costly liquidations of the long-term asset. This liquidation of the long-term asset will create dead-weight losses. The losses from a system with incomplete markets can be worse than the losses from unconnected banks[8]. This is a representation of financial contagion[9]. Eisenberg and Noe (2001) similarly consider cascading sequential default between linked institutions and show that as long as there are complete markets, a "clearing payment vector" will exist. There is no central authority in either of these models and the network formation is exogenous. For a comprehensive discussion of extensions to the Eisenberg and Noe model, see Glasserman and Young (2016).

The financial markets are made up of many kinds of actors of which banks are only one important class. Insurance companies, investment banks, hedge funds, mutual funds, pension funds, government agencies and non-financial corporations are also active financial market participants. We consider these other financial market participants, as a group, to be the NB[10]. The importance of these actors was highlighted by the failure of Long-Term Capital Management in 1998 and more recently of the mortgage companies and bank-sponsored structured investment vehicles during the sub-prime crisis of 2007. The central authority only directly regulates and supervises the banking sector. Hence, there is potential for regulatory arbitrage using risk transfers between sectors.

In this paper, we extend previous models of bank networks in three very important directions. First, we make the financial market network formation endogenous, such that the network of banks and NB is no longer fixed. We allow financial institutions to make bilateral links with each other and analyze the completeness, stability and social welfare of the financial system. Interestingly, under endogenous network formation, identical financial institutions may take on different roles with different welfare implications and we analyze the stability and social welfare characteristics of the resulting financial network. Second, we incorporate a social welfare-optimizing FR as an actor in the financial network. By creating an explicit role for the FR, we are able to examine the crucial role that regulatory policy can have on financial network formation. Finally, we focus on capital levels rather than liquidity as the key risk-sharing metric. In this way, we explicitly consider financial institution solvency as the critical condition, rather than looking at purely liquidity-driven events[11].

To do this, we rely on a different underlying architecture from Diamond and Dybvig (1983) and incorporate the literature of social and economic networks (Jackson and Wolinsky, 1996; Jackson and Watts, 2002). We assume that there are costs to establishing a connection and assume that individual firms form bilateral risk-sharing relationships[12].

Our model structure is based on the work of Bramoullé and Kranton (2005, 2007) but includes an asymmetric government regulator as a participant. In their model, individuals make bilateral transfers only if they have already formed a relationship that allows them to observe income levels and contract for risk sharing. They analyze equilibrium and efficient patterns of network relationships when pairs can agree to form links, but agents cannot coordinate link formation across the population. Hence, network formation in their model is a two-stage game. In the first stage, agents form links and then income shocks are realized and sharing occurs. A key difference in our model is to incorporate an FR as the center of the star-shaped network (Goeree *et al.*, 2009). In general, we find that efficient socially optimal risk-sharing networks will connect all financial market participants with high risk aversion (at least indirectly) and create full insurance among them. The regulator plays a critical role in financial network formation by setting the regulatory policy variables.

To find the equilibrium networks, we also apply and extend the notion of pairwise stability (Jackson and Wolinsky, 1996). In their model, the equilibrium markets formed bilaterally may connect fewer participants than would be socially optimal. This is because of an externality on bilateral risk sharing. Individuals that form networks only consider their own welfare and do not take into consideration the welfare of other participants. Babus (2006) has an endogenous extension of Allen and Gale (2000) that has a similar result.

After the network is established, each financial institution in our paper receives an idiosyncratic shock to its capital stock. For institutions that are members of the FR network, shocks are shared through the links of the network[13].

3. The model

We construct a benchmark model in which a country has one FR and m additional financial institutions that choose to become MBs, BHCS or NB. MBs are the financial institutions that are directly linked to the FR. BHCS are those that are not directly linked to the FR, but are directly linked to the MBs. NB are those that are singletons, which means that they are not linked to any of the other institutions. We usually assume that NBs do not link with each other without any of them being linked to the FR, but this possibility does come up as a limiting case in some of our results.

There are three stages in the game of network formation. In the first stage, some financial institutions link with the FR to become the MBs. In the second stage, other financial institutions can not only continue to become additional MBs, but they can also choose to become BHCSs by linking with existing MBs. If a financial institution chooses not to be linked with any other financial institution, it is a singleton. In the third stage, the credit shocks are realized. Then the participating MBs and BHCSs in the risk-sharing network have to share their idiosyncratic shocks via monetary transfers based on their *ex ante* agreement. One of the roles of the FR is to coordinate and facilitate such transfers.

When two financial institutions form a link, there is a linking cost incurred by each of them. Although financial institutions in the network share their idiosyncratic shocks, the costs of establishing a link cannot be shared. Hence, two financial institutions will form a link with each other only if both will be (strictly) better off than when no link is formed between them. This is referred to as the pairwise stability condition in the literature of network formation. However, if a linked financial institution intends to sever its link with others, only a unilateral decision is needed, although there is a cost incurred for unilaterally breaking a link[14].

We can make an analogy here to mergers, where the costs of merging are borne by each side, while the benefits are shared once the combination takes effect. Benefits of joining the FR network include forms of coinsurance and economies of scale. In particular, the ability to offer customers insured deposits and the ability to borrow from the Federal Reserve (and perhaps to be bailed out if necessary) are the principal benefits.

3.1 Risk averse utility

Assume that each financial institution's equity capital is initially $\bar{e}$. Once an idiosyncratic (credit) shock, ε_i, occurs, the capital of institution i will be $e_i = \bar{e} + \varepsilon_i$. All the idiosyncratic shocks are i.i.d. with mean zero and variance σ^2. All the m financial institutions before network formation have the utility function $u(e_i) = e_i - \lambda_i e_i^2$, with $\lambda_i > 0$ and $e_i < (1/2\lambda_i)$[15]. Such an assumption makes the utility functions increasing and concave, and at the same time implies that all the financial institutions are risk averse. The absolute risk aversion is $2\lambda_i/(1-2\lambda_i e_i)$, which is increasing in λ_i and in e_i. A lower λ_i means that a financial institution will be willing to take higher risks in its investments. (In the extreme case, a financial institution would be risk neutral if $\lambda_i = 0$.) Also, we assume that there are two types of financial institutions, one with higher $\lambda_i = \lambda_H$, and the other one with lower $\lambda_i = \lambda_L$.

One reason for risk aversion in financial institutions may be the position of the managers. Outside shareholders have the opportunity to diversify their portfolios. However, bank managers, who may be compensated in part with stock or options with long vesting periods, cannot be as diversified. As a result, they may manage the bank in such a way as to reduce risk, for example by limiting the number of risky loans that are made to customers.

Following Bramoullé and Kranton (2007), we assume that when networks are formed, institutions will maximize their utility net of any fixed linking costs (or benefits). Thus, we will define an overall utility function $U(e) = u(e)$−fixed costs.

3.2 The first stage

When a financial institution forms a link with the FR (becomes an MB), it incurs both fixed and variable costs. The cost, c, is interpreted as the one-time fixed cost of setting up the systems required for being regulated and monitored by the FR. This cost is paid by each individual institution that joins the network. The cost for the FR to form a link is assumed to be 0, since it already has its monitoring systems in place[16]. In addition, when a financial institution becomes an MB, the regulations imposed by the FR force it to invest more conservatively. For such MBs, λ_i becomes λ_M with $\lambda_M > \lambda_H > \lambda_L$. This is a variable cost of forming the link.

Assume that at the beginning of the first stage, all financial institutions are singletons. Then n out of m singletons decide to become an MB by linking with the FR, and thus indirectly to link with each other[17]. When equity shocks occur, they will be shared through these links, so for each of these MBs of the FR, the equity capital will be given by:

$$e_i = \bar{e} + \frac{\sum_{i \in N} \varepsilon_i}{n}, \quad \forall i \in N,$$

where $N = \{1, 2, \ldots, n\} \subset M = \{1, 2, \ldots, m\}$. Using the i.i.d. assumption about the shocks, and taking the linking costs into account, we can directly calculate the expected utility for an MB in the network as:

$$E(U(e_i)) = u(\bar{e}) - \frac{\lambda_M \sigma^2}{n} - c, \quad \forall i \in N.$$

Note that the expected utility function is an increasing, concave function of n. The increase in expected utility results from the increased risk sharing. Because this increase occurs at a decreasing rate, at some point it may be better for a financial institution not to become an MB because of the variable cost implicit in the higher required λ.

3.3 The second stage

In the second stage, other financial institutions can choose to join the network as additional MBs or to make themselves available to become BHCSs. If an institution links with an MB to become a BHCS, it incurs a fixed linking cost C, with $C > c > 0$. C is interpreted as the cost of negotiating the *ex ante* (before the credit shocks are realized) risk-sharing contract between the financial institutions, which in this case involves a merger. It represents the total cost paid by both sides. We assume that the negotiation cost is much lower to become an MB because the contract to become an MB is standardized. Hence, it is more costly for financial institutions to form links with each other than with the FR. However, these BHCSs can keep their original λ_i because they are not directly linked with the FR, and are not subject to the regulation of the FR. Thus, becoming a BHCS involves a higher fixed cost, but no variable costs[18]. If we allowed several NBs to link with each other first, before one of them linked with a BHC (or an MB), their expected utilities minus cost would be the same as if they become BHCSs. For reasons of simplicity, we rule out such a possibility. Therefore, in this paper, NBs do not form links with each other except in the limiting case where there are no MBs. They are either BHCSs or singletons that are not linked with any other financial institution.

Specifically, after the first stage, there are $m - n$ financial institutions that are not MBs. We assume that the number of these institutions that make themselves available for linking is large enough that each MB_i can choose the optimal number k_i (positive or 0) of BHCSs to maximize its individual expected utility[19], without the supply of available financial institutions being exhausted. Let $k = \sum_{i=1}^{n} k_i \leqslant m - n$ be the total number of BHCSs chosen by all the MBs. The expected utilities for these BHCSs are:

$$E(U(e_i)) = u(\bar{e}) - \frac{\lambda_i \sigma^2}{n+k} - C, \quad \forall i \in K,$$

where $K = \{n+1, n+2, \ldots, n+k\} \subset M \backslash N$.

We also assume that an MB gets a special benefit, B, with $B < C$, for each financial institution that it accepts as a BHCS. The underlying assumption is that there are equity transfers between the MB and its BHCSs. Because of this, an MB, restricted by the λ_M, can indirectly benefit from a higher expected rate of return through its BHCSs that have lower λ_i's. At the same time, a BHCS of an MB can also benefit from lower risk of investment through the link with the MB and thus the financial network.

Assume that $\sum_{i=1}^{n} k_i = k = k_i + k_{-i}$, where $k_{-i} = \sum_{j \neq i} k_j$. Thus, the ith MB links with k_i unlinked institutions and does not link to k_{-i} institutions. Note, however, that it is indirectly linked to all BHCSs through the FR. It then has the following expected utility:

$$E(U(e_i)) = u(\bar{e}) - \frac{\lambda_M \sigma^2}{n+k} - c - k_i C + k_i B, \quad \forall i \in N.$$

The MB wants to choose k_i to maximize this utility. The Kuhn-Tucker (K-T) conditions for this maximization are:

$$\begin{aligned} \frac{\partial E(U(e_i))}{\partial k_i} &= \frac{\lambda_M \sigma^2}{(n+k)^2} - C + B \leqslant 0 \\ (k_i)\left[\frac{\lambda_M \sigma^2}{(n+k)^2} - C + B\right] &= 0 \quad k_i \geqslant 0. \end{aligned} \tag{1}$$

P1. BHCS acceptance condition: an MB will link to a positive number of BHCSs if and only if the following condition is satisfied:

$$\frac{\lambda_M \sigma^2}{(C-B)} = (n+k)^2. \tag{2}$$

Proof. Utility will be maximized with a positive number of BHCS if there is an interior solution to (1), $k_i^* > 0$. This is true if the inequality on the first line of (1) is equality, i.e.:

$$\frac{\lambda_M \sigma^2}{(n+k)^2} = (C-B),$$

which is equivalent to (2). ■

Note that the derivative on the first line of (1) is decreasing in k_i. It follows that for any other k_i such that $0 \leqslant k_i < k_i^*$, $(\partial E(U(e_i))/\partial k_i) > 0$, which means that the expected utility is an increasing function of k_i, with $k_i < k_i^*$. In this case, even if an MB cannot get k_i^* BHCSs, the MB should still have as many BHCSs as possible. This is important because all n and k_is (and k) are integers. If the optimal solution from (1) is not an integer, a closest integer will be the solution. This also applies to all the other conditions in the paper.

In difficult times for the financial system, as in 2008, σ^2 will get larger, representing increased uncertainty. From (2), we see that this means that n and k will also increase. Thus, there will be more institutions joining the network.

Given $\lambda_M\sigma^2$, if there are enough MBs and k_{-i} BHCSs, it is possible that other financial institutions in the second stage will choose not to have any BHCSs:

P2. BHCS rejection condition: an MB will link to no BHCSs if the following condition is satisfied:

$$\frac{\lambda_M \sigma^2}{(n+k_{-i}+k_i)^2} - (C-B) < 0, \text{ for any } k_i \geqslant 0. \tag{3}$$

Proof. When each MB decides whether to take on BHCSs, some other MBs may already have done so. If $(n + k_{-i})$ is already sufficiently large that (3) holds, then the K-T condition implies that the optimal solution is the corner solution $k_i^* = 0$. ■

Condition (3) essentially says that the marginal costs of accepting additional BHCS will be greater than the marginal utility. *C–B* represents a fixed risk-sharing (diversification) benefit. In the 2008 situation, Merrill Lynch was the last BHCS accepted, in this case by Bank of America. Barclay's refused to buy Lehman Brothers before the bankruptcy.

Furthermore, if there are enough (or too many) MBs in the first stage, there will be no BHCSs at all in the second stage:

P3. No BHCS condition: no MB will link to any BHCS if the following condition is satisfied:

$$\frac{\lambda_M \sigma^2}{n^2} - (C-B) < 0. \tag{4}$$

Proof. If n is already sufficiently large such that (4) holds, then $(\lambda_M\sigma^2/(n + k)^2) - (C-B) < 0$, for any $k \geqslant 0$, so the K-T conditions imply that the optimal solution is $k_i^* = 0$, for every k_i.■

Note, however, that the possibility of satisfying these conditions is affected by the fact that $(n + k)$ is bounded above by m, the number of financial institutions available[20].

For the financial institutions that choose to be singletons, their expected utility functions are:

$$E(U(e_i)) = u(\bar{e}) - \lambda_i \sigma^2, \quad \forall i \in L,$$

where $L = M \backslash N \backslash K = \{n + k + 1, n + k + 2, \ldots, n + k + l = m\}$.

4. Network analysis

4.1 Endogenous network formation

There are different possibilities of endogenous *ex ante* network formation, which depend on the values of the parameters, and the numbers of participating financial institutions in each stage. The FR's main policy variable is λ_M which can be thought of as a proxy for minimum reserve and capital requirement. These are continuing constraints which were increased by Basel III and the Dodd-Frank Act. Most of the other parameters are fixed, but we assume that the FR can also have an effect on the linking cost c, which may be affected by FR behavior and/or by offsetting benefits that the FR can offer. This one-time cost is related to requirements for maintaining the information for required disclosures and carrying out the required filings.

We assume that $\lambda_L\sigma^2 < c \leqslant \lambda_H\sigma^2$:

P4. MB participation condition: financial institution i will choose to become an MB if and only if the following condition is satisfied:

$$\frac{\lambda_M\sigma^2}{n}+c\leqslant\lambda_i\sigma^2. \tag{5}$$

Proof. In the first stage, the necessary conditions for the n financial institutions to become MBs are for them to be better off as MBs than as singletons. (When one of them will be equally well off either way, we assume the financial institution will become an MB.) This means that:

$$\forall i\in N,\ u(\bar{e})-\lambda_i\sigma^2\leqslant u(\bar{e})-\frac{\lambda_M\sigma^2}{n}-c.$$

This is equivalent to (5). ■

Higher risk, σ^2, makes it more likely that (5) will hold, since $\lambda_i \geqslant \lambda_M/n$ for any reasonable n. The high risks in 2008 explain the decision of Goldman Sachs and Morgan Stanley to become MBs. In general, the unaffiliated NBs are the institutions with low risk aversion or ones that perceive low risk. In this case, we saw some independent investment banks that did not have large trading positions stay as NBs, like Oppenheimer or Wasserstein Perella.

The following corollaries are more fully developed in Appendix 2:

Corollary 1. All financial institutions with $\lambda_i=\lambda_L$ will remain singletons.

Corollary 2. All financial institutions with $\lambda_i=\lambda_H$ will want to join the network provided that $n\geqslant\bar{n}=\left(\lambda_M\sigma^2/\left(\lambda_H\sigma^2-c\right)\right)$[21].

P5. BHCS participation conditions: given $n\geqslant\bar{n}$, an NB with $\lambda_i=\lambda_H$ will choose to become a BHCS if the following two conditions are satisfied:

$$\frac{\lambda_H\sigma^2}{n+k}+C\leqslant\lambda_H\sigma^2, \tag{6a}$$

and:

$$\frac{\lambda_H\sigma^2}{n+k}+C\leqslant\frac{\lambda_M\sigma^2}{n+k}+c. \tag{6b}$$

Proof. An NB with $\lambda_i=\lambda_H$ has an expected utility of $U(e)=u(\bar{e})-\lambda_H\sigma^2$. An MB has expected utility of $U(e)=u(\bar{e})-\left(\lambda_M\sigma^2/(n+k)\right)-c$. A BHCS has expected utility of $U(e)=u(\bar{e})-\left(\lambda_H\sigma^2/(n+k)\right)-C$. An institution will choose to become a BHCS if the third expression is larger than the first two, which is equivalent to the above expressions. (6a) says that being a BHCS is better than being an NB, while (6b) says that being a BHCS is better than being an MB. ■

In 2008, Merrill Lynch chose to become a BHCS, in a relatively low cost acquisition by Bank of America. Similar to condition (5), conditions (6a) and (6b) mean that for a given $n\geqslant\bar{n}$, a singleton will become a BHCS only if there are enough of them to do so at the same time. However, if $C>\lambda_H\sigma^2$, then (6a) will never be satisfied and all institutions will remain singletons at stage 2. Hence, we assume that $\lambda_H\sigma^2 > C > c$:

P6. Bounds on the number of BHCS: assume that $n\geqslant\bar{n}$, but $n<\sigma\sqrt{\lambda_M/(C-B)}$ so that the no BHCS condition, (4), is not satisfied. If

$$\frac{\lambda_M\sigma^2}{n}+c\leqslant\frac{\lambda_H\sigma^2}{n}+C \tag{7}$$

holds, then all financial institutions with $\lambda_i = \lambda_H$ will become MBs instead of BHCSs, so $k=0$. If, instead:

$$\frac{\lambda_M \sigma^2}{n} + c > \frac{\lambda_H \sigma^2}{n} + C \tag{8}$$

holds, then:

$$\max\left(\frac{\lambda_H \sigma^2}{\lambda_H \sigma^2 - C}, \frac{\lambda_M \sigma^2}{\lambda_H \sigma^2 - c}\right) \leqslant (n+k) < \frac{\lambda_M \sigma^2 - \lambda_H \sigma^2}{C - c}.$$

Proof. If (7) holds, then the cost of being a BHCS is larger than the cost of being an MB, so no financial institutions will choose to become a BHCS. Now assume that (8) holds. The first lower bound on $(n + k)$ follows immediately from (6a), while the second follows from $n+k \geqslant n \geqslant \bar{n}$. The upper bound follows from (6b). From (8), (6b) holds when $k=0$. Note that both sides of (6b) are decreasing functions of k, since:

$$\frac{\partial\left(\lambda_M \sigma^2/(n+k) + c\right)}{\partial k} = \frac{-\lambda_M \sigma^2}{(n+k)^2} < 0,$$

and:

$$\frac{\partial\left(\lambda_H \sigma^2/(n+k) + C\right)}{\partial k} = \frac{-\lambda_H \sigma^2}{(n+k)^2} < 0.$$

Since $\lambda_M > \lambda_H$, the RHS of (6b). decreases faster than the LHS of (6b), when k increases. Therefore, if k is large enough, (6b) will no longer hold. (6b) can be rearranged to give the upper bound on $(n + k)$ and therefore on k. ■

The upper bound on $(n + k)$ implies that there is a limitation on how many singletons can become BHCSs. If additional financial institutions joining the financial network would violate condition (6b), the newcomers will only be able to become MBs. Thus, there are limits to the number of BHCS an MB will find acceptable. For example, Citigroup spun off Travelers in 2002 when they got too big.

Now, we consider how the magnitude of the upper bound, $((\lambda_M\sigma^2 - \lambda_H\sigma^2)/(C-c))$, depends on its parameters. First, the larger C is relative to c, the smaller the number k of financial institutions that will become BHCSs because it is more costly for them to do so. Second, the larger the difference $(\lambda_M - \lambda_H)$, the larger the number k can be. This means more financial institutions will become BHCSs if the FR raises λ_M, and fewer singletons will be interested in doing so if the FR lowers the λ_M. Third, the larger the variance, σ^2, the larger the number k can be.

At the same time, because the formation of a link requires agreement of both sides, it is equally important that the MBs are willing to accept BHCSs:

P7. Pairwise stability of an MB–BHCS link: an MB will accept one NB as a BHCS only if:

$$\frac{\lambda_M \sigma^2}{n + k_{-i} + 1} + (C - B) < \frac{\lambda_M \sigma^2}{n + k_{-i}}.$$

Proof. From the K-T conditions, (1), for a given number $n \geqslant \bar{n}$ of MBs and $k_{-i} \geqslant 0$ of other BHCSs, an MB will accept $k_i \geqslant 1$ financial institutions to become its BHCSs only if:

$$\frac{\lambda_M \sigma^2}{n + k_{-i} + k_i} + c + k_i C - k_i B < \frac{\lambda_M \sigma^2}{n + k_{-i}} + c. \tag{9}$$

If we had $k_i = 0$, then (9) would be an equality, so we must have $k_i > 0$, meaning that there is an interior solution of (1) satisfying: $(\lambda_M\sigma^2/(C-B)) = (n + k)^2$. Substituting $k_i = 1$, into (9) we get the desired result. ■

Pairwise stability is represented by the requirement that both sets of shareholders agree to a merger, as was the case for the merger of Merrill Lynch and Bank of America.

4.2 Stability conditions for the network

From the definitions of Jackson and Wolinsky (1996), there are two conditions for the stability of a network. In the context of our financial network, the first stability condition means that any linked financial institution, MB or BHCS, cannot be strictly better off by unilaterally severing its link. The second stability condition means that any two unlinked financial institutions cannot both be strictly better off by establishing a link.

The first stability condition is also related to the cost of breaking a link. An MB or BHCS that unilaterally breaks a link has to bear the cost. Assume that the cost to break a link between an MB and the FR is D, and the cost to break a link between a BHCS and an MB is d. We make no assumption here about the relative sizes of D and d. If an MB wants to drop its bank charter to become an NB, it will definitely attract serious scrutiny from the FR, which will contribute to the cost D. On the other hand, once you are a BHCS, you cannot just decide to become independent. You have to go through a costly spin-off process, resulting in the cost d. Which of the two processes is more expensive is not clear without some additional assumptions that we do not need here:

P8. Network stability: under the assumptions made in the previous propositions, all the pairwise stability conditions hold, and the network consisting of n MBs, $k \geqslant 0$ BHCSs and l NBs as singletons is stable.

The proof of *P8* is given in Appendix 4.

5. Efficient network vs stable equilibrium network

5.1 Efficient network

An efficient network is one that maximizes the aggregate expected utilities subtracting all the linking costs of all the financial institutions in the economy. Assume that there are $n \geqslant 0$ MBs, $k \geqslant 0$ BHCSs and $l > 0$ NBs as singletons in the economy, where l is given but n and k are endogenous. However, regardless of how or whether financial institutions with $\lambda_i = \lambda_H$ choose to become MBs or BHCSs, eventually, $(n + k) = (m - l)$ must hold. Therefore, there is only one free parameter to be determined here, n or k. The only singletons are the NBs with $\lambda_i = \lambda_L$. The aggregate expected utilities minus all the linking costs are:

$$\sum_{i \in N}\left[u(\bar{e}) - \frac{\lambda_M \sigma^2}{n+k} - c\right] - nc - k(C-B) + \sum_{i \in K}\left[u(\bar{e}) - \frac{\lambda_H \sigma^2}{n+k}\right] - kC + \sum_{i \in L}\left[u(\bar{e}) - \lambda_L \sigma^2\right].$$

Note that all terms not involving n or k are constant. For the FR, maximizing the aggregate utilities minus all the linking costs is equivalent to minimizing the shared risks and linking costs, which are all negative in the above expression:

$$\min_{n \geqslant 0, k \geqslant 0} A \equiv n\left[\frac{\lambda_M \sigma^2}{n+k}\right] + nc + k\left[\frac{\lambda_H \sigma^2}{n+k}\right] + k(2C - B),$$

subject to $n + k = m - l$ and $l > 0$:

P9. Efficient network: in the efficient network, there may be no MBs, there may be no BHCSs or else all choices of the numbers of MBs and BHCSs are equally good from the point of view of the FR.

Proof. Assume that k is the choice variable of the FR. Then A is linear in k (since $n+k$ is constant), so the social optimum will be at one of the two possible corner solutions. We have, using $n=(m-l)-k$:

$$\frac{\partial A}{\partial k}=-\left[\frac{\lambda_M\sigma^2}{m-l}\right]-c+\left[\frac{\lambda_H\sigma^2}{m-l}\right]+2C-B=\left(\frac{-1}{m-l}\right)\left[(\lambda_M-\lambda_H)\sigma^2+(m-l)(B+c-2C)\right].$$

If $(\partial A/\partial k)>0$, which means $(\lambda_M-\lambda_H)\sigma^2<(m-l)(2C-B-c)$, then to minimize A, we need $k=0$, and $n=(m-l)$, which is the first corner solution. This implies that if the FR sufficiently lowers λ_M, σ^2 is small, and/or $(2C-B-c)$ is sufficiently large, all the financial institutions with $\lambda_i=\lambda_H$ should become MBs, as long as there are enough financial institutions to allow $n > (((\lambda_M-\lambda_H)\sigma^2)/(2C-B-c))$. This is the complete financial network in which all financial institutions with $\lambda_i=\lambda_H$ become MBs, while the NBs with $\lambda_i=\lambda_L$ institutions remain as singletons.

If $(\partial A/\partial k)<0$, which means $(\lambda_M-\lambda_H)\sigma^2 > (m-l)(2C-B-c)$, then to minimize A, we need $k=(m-l)$ and $n=0$, which is the second corner solution. This implies that if the FR sufficiently increases λ_M, σ^2 is large and/or $(2C-B-c)$ is sufficiently small, there will be no MBs. Although we have been assuming that there are no links between non-banks, in this case it would be optimal for all the financial institutions with $\lambda_i=\lambda_H$ to form complete risk-sharing networks among themselves with $k < (((\lambda_M-\lambda_H)\sigma^2)/(2C-B-c))$. Note that because there are l NBs with $\lambda_i=\lambda_L$, an efficient network does not always have to include all the financial institutions in the economy, and it is possible for NBs to exist as singletons.

If $(\partial A/\partial k)=0$, then $(\lambda_M-\lambda_H)\sigma^2=(m-l)(2C-B-c)$. In this case, the FR is actually indifferent between the numbers of MBs and BHCSs, as long as $(n+k)=(((\lambda_M-\lambda_H)\sigma^2)/(2C-B-c))$. Although the numbers n and k are indeterminate individually, if $(\lambda_M-\lambda_H)$ and/or σ^2 is large, more financial institutions will join the network. In contrast, if $(2C-B-c)$ increases, fewer financial institutions will participate in the financial network. In addition, if we combine this result with (1), it implies that:

$$n+k=\sqrt{\frac{\lambda_M\sigma^2}{C-B}}=\frac{(\lambda_M-\lambda_H)\sigma^2}{2C-B-c}$$

should hold. In this case, the FR lets the financial institutions choose the numbers of MBs and BHCSs by themselves, but the FR may be able to set the policy variables to choose the total number of MBs and BHCSs in the network. ■

Suppose we want to allow for the existence of some NBs with $\lambda=\lambda_H$: some financial institutions with $\lambda=\lambda_H$ may not become either MBs or BHCSs. This would mean that we have inequality in the feasibility constraint, $(n+k)\leqslant(m-l)$. Could there be any interior solutions? Such solutions would have to satisfy $(\partial A/\partial n)=(\partial A/\partial k)=0$. But:

$$\frac{\partial A}{\partial n}=k\frac{\lambda_M\sigma^2}{n+k}+c-k\frac{\lambda_H\sigma^2}{(n+k)^2}=(n+k)^{-2}\left[k(\lambda_M-\lambda_H)\sigma^2\right]+c>0,$$

and:

$$\frac{\partial A}{\partial k}=-n\frac{\lambda_M\sigma^2}{(n+k)^2}+n\frac{\lambda_H\sigma^2}{(n+k)^2}+2C-B=(n+k)^{-2}\left[n(\lambda_H-\lambda_M)\sigma^2\right]+2C-B,$$

which is positive for small n and negative for large n, so there are no interior solutions. Therefore, all financial institutions with $\lambda=\lambda_H$ will become either MBs or BHCSs in the efficient network.

5.2 Stable equilibrium network

The stable equilibrium network, however, is not always the efficient network. In Section 4.1, we have shown that the stable equilibrium network depends on the self-interests of individual financial institutions. Each financial institution considers only its own expected utility (minus the cost), and not the externality of its effects on risk sharing with other (more distant) members of the network. Whether an efficient network can be formed is not its concern. The singletons want to become BHCSs because they do not want to become MBs and have to change the risk levels of their investments. At the same time, becoming BHCSs will give them lower risk because of diversification, even though the linking costs are higher than the linking costs of becoming MBs. For the financial institutions that become the early MBs, they also want to lower their risk of investment by choosing the optimal number of BHCSs, if the sufficient condition holds. If the condition does not hold, then it is also likely that more or all of the $(m-l)$ financial institutions with $\lambda_i = \lambda_H$ will become MBs.

In addition, when the stable equilibrium network is not the efficient network, the FR will act to move them closer. For example, in the third case of the efficient network:

$$(n+k) = \frac{(\lambda_M - \lambda_H)\sigma^2}{2C-B-c} = \frac{(\lambda_M - \lambda_H)\sigma^2}{(C-B)+(C-c)}$$

is still less than the upper bound derived from (8), i.e., $(n + k) \leqslant ((\lambda_M\sigma^2 - \lambda_H\sigma^2)/(C-c))$. Therefore, one of the important roles of the FR is to set its policy variables λ_M and c to make sure that the stable equilibrium network can coincide with or be as close as possible to the efficient network. Note that there may be more than one stable network. In particular, if we assume that no institution will want to change its status to a different status with the same utility, then the efficient network will be stable.

Only the FR considers the efficiency of the system. The fact that financial institutions whether MBs or BHCSs are myopic and only consider their own situations helps to explain how competitive markets drove the system instability that became the Great Recession.

6. Conclusion and extensions

The pairwise deal making of the financial market participants, MBs, BHCS and other NB entities is fundamentally myopic. The financial market participants do not consider any of the social benefits from longer or more complete networks by themselves. Therefore, it seems to be an appropriate role for the socially beneficent central governmental regulator to use their policy tools, financial benefits and regulatory burdens, to help drive the equilibrium outcome toward the socially optimal efficient outcome, which typically includes more risk-sharing connections. In this model, there are two policy tools. The first policy and model variable is the cost, c, for a financial institution to become an MB, i.e., to link to the FR. The second model variable is λ_M, the risk aversion of an MB. The FR controls this indirectly by policy choices such as capital requirements. Because of frictions due to the cost of breaking links, it is possible that the stable equilibrium network has more members than the efficient one. Other constrained or corner solutions are difficult to generalize so the regulatory challenge is very difficult to optimize. In the real world, we observe that the Federal Reserve Board recruits a diverse group of experts who make incremental shifts and carefully observe market conditions over time, which may be their strategy to help make sure that the efficient network coincides with or is as close as possible to the stable equilibrium network. This may represent a dynamic solution to our simplified model.

As this paper builds an *ex ante* risk-sharing financial network, there are many directions for subsequent extensions. Here are five of them.

First, we would like to undertake development of a new measure of the distance between the equilibrium and socially efficient solutions. This would allow us to develop an objective function that would allow us to consider how the governmental central regulator could modify and relax the assumptions of our model. This will allow us to consider how a member of the network will be able to make changes that result in utility changes and consider the stability of the efficient solutions. We would also be able to consider whether there may be multiple stable equilibria and evaluate their relative efficiency.

Second, in this paper, we focus on the *ex ante* expected utility functions, but equally important is the *ex post* analysis after all the idiosyncratic shocks are realized. Under an *ex post* analysis, we could have idiosyncratic shocks causing bankruptcies among the singleton NBs. If the NBs are therefore forced to liquidate and lower the price of a common equity asset, they will cause systemic shocks to the market. This is one direction we would like to explore.

Third, note that risk sharing among financial institutions can lower only the idiosyncratic shocks. If there are, however, systemic shocks, i.e., every MB and BHC will receive the same shock, then risk sharing will not be able to lower the risks of all the financial institutions in the network. There are two ways to resolve such a problem. One is FR intervention. In the other one, we incorporate the possibility of creating networks that link across regions (or countries). These linkages may occur between FRs and mimic the behavior of the G7 in coordinating policy after a shock. The coordination between the major FRs in response to the sub-prime crisis and its fallout is an example. This is another direction in which we could continue our research.

A fourth extension of our model is to allow the banks to have different equity capital stocks. In this case, a bank with larger equity may be more resilient to equity shocks. However, if larger equity comes from merging with other MBs or NBs, it may reduce the number of banks for diversification and increase the investment risk. This is another direction we would like to explore in the future.

Fifth, in this paper we focus on the situation when $C > B$ and assume that the financial institutions are not farsighted. If the financial institutions are farsighted and only care about the later (or final) expected utility, and there are two waves of financial institutions that join the network, then the first wave of financial institutions that become MBs can choose not to have any BHCSs at all. This would make the first wave of financial institutions temporarily worse off, but eventually better off because the financial institutions in the second wave will be forced to become MBs. The financial institutions in the first wave are better off because the investment risk is lower when there are more MBs. At the same time, they do not have to pay for the (net) linking cost because $C > B$. Therefore, when financial institutions are farsighted, MBs will accept financial institutions to be the BHCSs only if $C < B$, which is also consistent with the principles of net present value in finance. But since all the MBs are symmetric, it is likely that each one of them has the same number of BHCSs. The other possibility is that there could be some sort of discounting between the utilities in the first wave and in the second wave. If that is the case, even if B is slightly less than C, it may still be possible for the MBs to accept other financial institutions as BHCSs in the first wave. Therefore, the dynamic network formation with farsightedness and discounting is another direction that we would like to explore.

Notes

1. Dodd-Frank Wall Street Reform and Consumer Protection Act, Public Law 111-203, July 21, 2010. Text available at: www.gpo.gov/fdsys/pkg/PLAW-111publ203/pdf/PLAW-111publ203.pdf.

2. Real central banks also have the sometimes-conflicting goal of promoting economic growth and full employment.

3. Beyhaghi *et al.* (2017) describe banks choosing between credit derivatives and secondary loan sales to manage credit risk. Much of the literature on financial networks deals with the case when links represent loans from one financial institution to another rather than corporate or regulatory relationships. We discuss more of this literature in the next section.

4. Sometimes banks are assumed to be risk neutral, but risk aversion is certainly a common assumption, for which several justifications have been suggested. There is an extensive literature on this issue. See the survey of Santomero (1984). Angelini (2000) provides some empirical evidence for bank risk aversion.
5. As Gallegati *et al.* (2008) point out that, in a bad economic environment, risk sharing can also be detrimental.
6. In the real world, the occurrence of shocks often leads to changes in the network structure through failures and mergers, as occurred during the 2008 financial crisis.
7. The member banks play symmetric roles in the model. Although they receive independent shocks, they are i.i.d.
8. Allen and Gale (2005) also considered the issues of risk allocation between banks and insurance companies and found that regulation could be justified when there are incomplete markets.
9. Brusco and Castiglionesi (2007) also model financial contagion across linked regions.
10. Many of these are now commonly referred to as shadow banks.
11. Recent government actions to stress-test large banks involve the effects of worst-case scenarios on capital adequacy and solvency.
12. In the context of risk sharing in networks of individuals, empirical research in the rural Philippines shows that when shocks occur, households receive support from networks of friends and relatives rather than from formal village structures (Fafchamps and Lund, 2003).
13. In an extension to the basic model, disconnected non-banks may become insolvent and be forced into "fire-sale" liquidation. This forced liquidation lowers the trading value of the common asset in which financial institutions' capital is invested. The reduced price causes a systemic capital reduction and propagates the shock to other financial institutions. It may cause cascading contagion. Eboli (2007) and Nier *et al.* (2007) use a similar mechanism to create systemic shocks in financial networks. Timely policy action by the central bank to inject offsetting funds can stop the spillover and eliminate the contagion risk.
14. In practice, member banks do not stop being member banks except when they fail. In this case, the cost includes the loss to the equity holders as well as the administrative costs of liquidation or sale to another institution. For a BHCS, on the other hand, the link is broken when the subsidiary is sold or spun off. This could actually be profitable, i.e. the cost could be negative.
15. Santomero (1984) says that the literature usually uses quadratic or exponential utility (p. 582).
16. There may be a marginal cost for the FR to monitor an additional MB, but we can assume that since it is the function of the FR to do this, they will always accept new MBs regardless of cost, assuming the MB meets minimum regulatory requirements for holding a banking charter. Our focus here is on the decisions of the financial institutions other than the FR.
17. One may ask, when the first institution decides to become an MB, how does it know how many others will also do so. We assume that the FR starts by specifying a minimum number such that no institution will become an MB until that number have signed up to do so, and then they will all join simultaneously.
18. Note that if it were the case that $C < c$, then institutions would prefer to form links among themselves without any connection to the FR.
19. From the model specifications, it appears that all the MBs are solving the same problem here. We assume that there are differences among them that are not explicitly modeled that can result in their choosing different k_is. For example, an MB in a geographical region with higher population density might be likely to choose a larger k_i. These choices are made sequentially so that each MB chooses to link to BHCSs from the institutions that have not already linked with other MBs that chose earlier.
20. There may be a free-rider problem here. An MB might prefer to choose last in stage 2. Let the other MBs take on the BHCSs so that the one choosing last may get the benefit of the larger

denominator in the first term without the costs in the second term. But then no MB will take on any BHCS ($k_i = 0$ for all i) and everyone loses, a sort of prisoner's dilemma. Enforcing an order of selection may mitigate this problem.

21. Note that $\bar{n}$ may not be an integer.

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Appendix 1. A numerical example

In this appendix, we provide a numerical example with the following parameters. Suppose $\lambda_M = 0.005$, $\lambda_H = 0.0025$ and $\lambda_L = 0.00125$. $\bar{e} = 47$, and ε_i is a random variable from a continuous uniform distribution on [−48, 48]. It follows that $\sigma^2 = (96^2/12) = 768$ from the continuous uniform distribution. Hence, even if $\varepsilon_i = 48$, which makes $e_i = 47+48 = 95$, $e_i < (1/2\lambda_i)$ still holds.

In addition, assume that $C = 1.01$ and $c = 1.0$. From the information above, we can calculate $\lambda_M\sigma^2 = 3.84$, $\lambda_H\sigma^2 = 1.92$ and $\lambda_L\sigma^2 = 0.96$. Thus, $\lambda_H\sigma^2 > C > c > \lambda_L\sigma^2$ is satisfied. Therefore, we have:

$$n \geqslant \bar{n} = \frac{\lambda_M \sigma^2}{\lambda_H \sigma^2 - c} = 4.17,$$

which means that $n \geqslant 5$. That is, the minimum number of financial institutions needed to become MBs is 5. At the same time, $((\lambda_H\sigma^2)/(\lambda_H\sigma^2 - C)) = 2.1$, which means if $(n + k) \geqslant 3$, financial institutions have incentive to become BHCSs. Therefore, as soon as $(n+k) \geqslant 5$, financial institutions can become either MBs or BHCSs.

At the same time, $((\lambda_M\sigma^2 - \lambda_H\sigma^2)/(C-c)) = 192$, which is the maximum value of $(n+k)$ from the BHCS participation condition (6b). Assume that there are 200 financial institutions (in addition to the FR), i.e., $m = 200$. Among them, assume that there are $l = 10$ with $\lambda_i = \lambda_L$, so that $m-l = 190$, which is the upper bound of $(n+k)$. Because $192 > 190$, $(n+k) = 190$.

Further, if $B = 1.0099$, then $B < C$ is satisfied, and $C - B = 0.0001$. But more importantly, from (1), $(\lambda_M\sigma^2/(C-B)) = 38{,}400 = (n+k)^2$, which implies that $(n+k) = 195.96 > 190$. Hence, $m - l = 190$ is the binding upper bound of $(n+k)$. If $n = 20$ in the first stage, then $k = 190 - 20 = 170$ in the second stage. And $k = 0$ in the second stage only if $n = 190$ in the first stage. If each MB has one BHCS, then $n = k = 95$. However, there is no guarantee that all the MBs have the same number of BHCSs. If each MB can have up to two BHCSs, then $n = 64$, and $k = 126$, which implies that one MB has no BHC. If each MB can have up to three BHCSs, then $n = 48$, and $k = 142$, which implies that 47 MBs have three BHCSs, while one MB has only one BHC. Also, in terms of the efficient network from the perspective of the FR, $(\lambda_M-\lambda_H)\sigma^2 = 1.92 > (m-l)(2C-B-c) = 1.919$, with $k < ((\lambda_M - \lambda_H)\sigma^2/(2C-B-c)) = 190.09$. Hence, the equilibrium network should be $k = 190$, the complete network connecting all the financial institutions with $\lambda_i = \lambda_H$ to each other, without any being connected to the FR. In this case, it will be difficult or impossible for the equilibrium network to be the efficient network.

If $B = 1.0098$, then $B < C$ is also satisfied. Also from (1), $(\lambda_M\sigma^2/(C-B)) = 19{,}200 = (n+k)^2$, which implies that $(n+k) = 138.56 < 190$. Hence, there will be a first wave of financial institutions becoming MBs and BHCSs, and then a second wave of financial institutions that become MBs. In this case, assume that one MB can have up to one BHC, then $n = k = 69$ will occur first. The other 52 financial institutions will also become MBs, but without BHCSs simply because it is no longer beneficial for MBs to accept any more BHCSs. Therefore, $n = 121$, and $k = 69$. In total, 52 MBs do not have BHCSs. Similarly, if $n = 138$ or higher in the first stage, then no MBs will accept other financial institutions to be BHCSs. That is, all 138 MBs will choose $k_i = 0$ and thus $k = 0$. Then all 52 other financial institutions will become MBs as well. In terms of the efficient network from the perspective of the FR, $(\lambda_M-\lambda_H)\sigma^2 = 1.92 < (m-l)(2C-B-c) = 1.938$, with $n > 188.235$. Hence, the equilibrium network should be $n = 190$, all the financial institutions with $\lambda_i = \lambda_H$ should become the MBs, and there should be no BHCSs. In this case, it is possible or easier for the efficient network to coincide with the equilibrium network.

Appendix 2. Corollaries to *P4*

We assume that there are $l \geqslant 1$ financial institutions, with $\lambda_i = \lambda_L$. Note that the left-hand side of (5) decreases to c asymptotically as n goes to infinity. Hence, we have:

Corollary 1. All financial institutions with $\lambda_i = \lambda_L$ will remain singletons.

For the other financial institutions, with $\lambda_i = \lambda_H$, under what conditions will they join the network? We note that the left-hand side of (5) is a decreasing function of n, and in this case is greater than the right-hand side when $n = 1$ since $\lambda_M > \lambda_H$. However, for large n, (5) will hold. Hence, in order for these financial institutions to become MBs, there have to be enough of them to do so at the same time:

Corollary 2. All financial institutions with $\lambda_i = \lambda_H$ will want to join the network provided that $n \geqslant \bar{n} = \left(\lambda_M\sigma^2/\left(\lambda_H\sigma^2 - c\right)\right)$ (see footnote 21).

Appendix 3. Boundary conditions for BHCS

Following *P7*, we have applied the K-T condition to derive the pairwise stability condition. This is the only pairwise stability condition that we need to check. For the other links, for example, between the MB and the FR, there is no need to check the pairwise stability condition for the FR, because it always accepts new members.

So far, we have three upper bounds for $(n+k)$. From the negation of (4), we have $(n+k) \leqslant \sqrt{\left(\lambda_M\sigma^2/(C-B)\right)} \equiv x$, which limits k to the maximum number of financial institutions that MBs will accept as BHCSs for a given n. From (8), we have $(n+k) < ((\lambda_M\sigma^2 - \lambda_H\sigma^2)/(C-c) \equiv y$, which limits k to the maximum number of financial institutions that are willing to become a BHCS instead of an MB for a given n. Since forming a link requires agreement from both sides, the constraint with the smaller RHS will be binding. All the other financial institutions with $\lambda_i = \lambda_H$ beyond the binding constraint will become MBs instead of BHCSs since a bilateral agreement is not possible. At the same time, we have $(n+k) \leqslant (m-l) \equiv z$ as a feasibility constraint. These three upper bounds for $(n+k)$ can be ordered in six possible ways. If $x<y<z$, $y<x<z$, $x<z<y$ or $y<z<x$, then the first wave of financial institutions forming MBs and BHCSs is bounded by the smaller of x and y, the smaller of the number of MBs or NBs who are willing to form an MB–BHCS link. Beyond that, the second wave of all the other financial institutions with $\lambda_i = \lambda_H$ will only become MBs up to the feasibility constraint. If $z<x<y$ or $z<y<x$, so that the feasibility constraint is the tightest, then there is only the first wave of financial institutions forming MBs and BHCSs up to the feasibility constraint with no second wave.

Nevertheless, in the second stage, financial institutions may have another choice, which is to become BHCSs. Suppose n financial institutions have already chosen to become MBs in the first stage, with $n \geqslant \bar{n}$. Then in the second stage, a singleton will choose to become a BHCS if it is better off that way than if it either becomes an MB or remains a singleton.

Appendix 4. Proof of *P8*

First, we consider the NBs that are singletons. There should be no incentive for any one of them to become a BHCS or an MB. This means that:

$$\lambda_L \sigma^2 < \frac{\lambda_L \sigma^2}{n+k+1} + C, \tag{A1}$$

and:

$$\lambda_L \sigma^2 < \frac{\lambda_M \sigma^2}{n+k+1} + c. \tag{A2}$$

Since we assume that $\lambda_L \sigma^2 < c < C$, both (A1) and (A2) are satisfied for any $(n+k) \geqslant 0$.

Second, none of the k BHCSs that are in the network should have an incentive to break the link with its MB and then link with the FR to become an additional MB. This will be true if:

$$\frac{\lambda_H \sigma^2}{n+k} + C < \frac{\lambda_M \sigma^2}{(n+1)+(k-1)} + c + d. \tag{A3}$$

(A3) is always satisfied if $C < c + d$. If $C > c + d$, then from the upper bound in *P6*, we have:

$$(n+k) < \frac{\lambda_M \sigma^2 - \lambda_H \sigma^2}{C-c} < \frac{\lambda_M \sigma^2 - \lambda_H \sigma^2}{C-(c+d)},$$

which gives us (A3). The existence of the link-breaking cost d may increase the number of BHCSs.

Third, none of the k BHCSs should have an incentive to break the link with its MB to become a singleton. This will be true if:

$$\frac{\lambda_H \sigma^2}{n+k} + C < \lambda_H \sigma^2 + d, \ \forall i \in K. \tag{A4}$$

If $C < d$, i.e., it is more costly to break the link with an MB than to form one, then (A4) is always satisfied. Even if $C \geqslant d$, from (6a) we have:

$$(n+k) \geqslant \frac{\lambda_H \sigma^2}{\lambda_H \sigma^2 - C} > \frac{\lambda_H \sigma^2}{\lambda_H \sigma^2 - C + d},$$

which gives us (A4).

Fourth, an MB with no BHCSs should not have an incentive to break its link with the FR to become a BHCS. This means that:

$$\frac{\lambda_M \sigma^2}{n+k} + c < \frac{\lambda_H \sigma^2}{(n-1)+(k+1)} + C + D. \tag{A5}$$

(A5) can be combined with the upper bound derived from (6b) to give:

$$\frac{\lambda_M\sigma^2-\lambda_H\sigma^2}{C-c+D}<(n+k)<\frac{\lambda_M\sigma^2-\lambda_H\sigma^2}{C-c}.$$

At the same time, if any other MB accepts such a former MB as a BHCS, the MB will be strictly worse off because:

$$\frac{\lambda_M\sigma^2}{n+k}+c<\frac{\lambda_M\sigma^2}{(n-1)+(k+1)}+c+(C-B).$$

Therefore, it is impossible for an MB to get a bilateral agreement with another MB to become a BHCS.

Finally, an MB with no BHCSs should not have an incentive to become a singleton. This will be true if:

$$\frac{\lambda_M\sigma^2}{n+k}+c<\lambda_H\sigma^2+D. \quad \text{(A6)}$$

But we have assumed that $n \geqslant \bar{n}$, so:

$$(n+k)\geqslant n\geqslant \bar{n}=\frac{\lambda_M\sigma^2}{\lambda_H\sigma^2-c}>\frac{\lambda_M\sigma^2}{\lambda_H\sigma^2-c+D},$$

which gives us (A6).

Since none of the financial institutions has an incentive to break an existing link or establish a new link, the network is stable.

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